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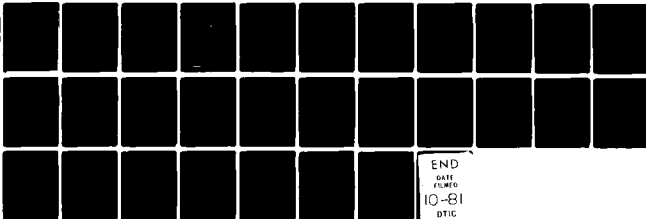
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The Economics of Military Capital

Lt. Col. Gregory G. Hildebrandt

August 1980

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Military capital is an aggregate measure of all durable assets of the defense establishment. The report investigates two related measures of military capital: the instantaneous-productive capacity of military capital, which equals the total monetary value of the benefits provided by the assets at some point in time, and the long-run productive capacity of military capital, which equals the monetary value of the benefits by the assets over the remainder of their service lives. These two measures can be determined by utilizing information that is likely to be available to the analyst. Both deterioration and changes in asset quality are properly accounted for in the determination. (Author)

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The Economics of Military Capital

Lt. Col. Gregory G. Hildebrandt

August 1980

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PREFACE

U.S. and Soviet military capital are important determinants of relative military capability. Therefore, careful measurement of the aggregate value of the durable assets in both military arsenals—the military capital stocks—would be helpful in monitoring the status of the long-term military competition.

This report investigates some conceptual aspects of military capital determination as part of the Project AIR FORCE study "Soviet Strategic Competitiveness: Constraints and Opportunities." Two measures of military capital are considered. One, the instantaneous productive capacity of military capital, is the monetary value of the services provided by the durable defense assets at a point in time. The second measure, the long-run productive capacity of military capital, is the monetary value of the services provided by the durable assets for the remainder of their service lives. Both of these measures are useful, although it is the instantaneous measure that reflects the contribution of the durable defense assets to military capability at some specific time.

The nature of this contribution is considered in a second report under this study: *Military Expenditure, Force Potential, and Relative Military Power*, R-2624-AF, August 1980.

SUMMARY

Military capital is a primary factor determining military capability. It is a summary measure of the aggregate value of the durable physical assets of the defense establishment and includes the military equipment, facilities, spare parts, and ordnance that provide military capability over an extended period.

This study investigates some conceptual issues associated with measuring military capital in monetary terms. Money is actually the most convenient yardstick for measuring military value. When a decisionmaker allocates defense resources efficiently, he reveals the monetary value of an additional unit of a military asset: the incremental benefits are equated to the incremental costs, and the costs are observable.

Section II defines two measures of military capital. One, the instantaneous productive capacity of military capital K_I , is the monetary value of the services provided by the durable physical assets at a given time. A second measure, the long-run productive capacity of capital K_L , is the monetary value of the services provided by the assets over the remainder of their service lives. Both measures have roles to play in the understanding of military capability. However, the instantaneous measure is probably a more useful summary statistic because it measures the contribution of the durable physical assets to the achievement of military force potential at a point in time.

Several functions are required to aggregate the diverse defense assets. The deterioration function describes the decline in relative efficiency as an asset ages, and the military effectiveness function describes the rise in relative efficiency as the quality level increases. These two functions are used to measure the relative efficiency of each asset and to weight the different capital elements in the aggregation process. For the instantaneous measure of capital, it is also necessary to know the monetary value of an additional unit of effectiveness provided at some time. A valuation function describes this monetary measure of military effectiveness. In contrast, for the long-run measure of capital, it is necessary to know the monetary value of the services provided by an asset over the remainder of its life. A second valuation function, called the demand price function, determines this value.

Methods are presented for identifying the two valuation functions and the two relative efficiency functions. Exponential deterioration is assumed (there is a discussion of straight-line deterioration in the appendix). Only the type of information likely to be available is required to identify the four key functions.

It is necessary to aggregate the capital elements within a particular class of military assets and also across the different asset types. Intra-asset aggregation is achieved because labor and material are optimally allocated to the units within an asset class. Inter-asset aggregation is achieved because the relative prices of the different asset types are assumed to be unchanging over time.

A policymaker who needs summary measures of the different dimensions of military capability, relative military power, and the status of long-term competition would do well to determine military capital measures.

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Abraham S. Becker, the project leader, provided guidance and counsel throughout the investigation. A substantial part of my one-year military assignment to Rand was performed under his direction, and he is largely responsible for making the year a success. Robert Shishko and Peter Stan carefully reviewed an earlier draft of this report, and their comments were perceptive as well as useful. Also Robert Shishko generously shared his files on capital theory and directed me to relevant literature. Ruthlouse Acar deserves special thanks for typing several versions of this analysis and making many helpful suggestions while remaining cheerful throughout numerous revisions.

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I. INTRODUCTION

The aggregate monetary value of all physical assets in the defense arsenal is called the military capital stock. These assets include military equipment (such as tanks and planes), military facilities (such as headquarters and supply depots), and the spare parts and ordnance that are held in inventory. All of these assets are durable inputs to the achievements of military capability; they are not fully consumed during their acquisition period and continue to provide military benefits throughout their service lives.¹

This study investigates some conceptual aspects of military capital determination. The emphasis is on the determination of methods of valuating and aggregating the various capital elements by exploring the following types of questions at a general level of discussion: What is the military (and monetary) value of all units of a particular type of military equipment such as the F-15 tactical fighter? How are all the members of this general asset type made commensurable so that their military values can be aggregated? How are these fighters and the other capital elements, including the tanks, missiles, submarines, facilities, etc., aggregated into an overall measure of military capital?

At least two measures of military capital can be considered. One is the *instantaneous productive capacity* of military capital, K_I . This measure summarizes the value of the services provided by the military capital stock at some point in time. The physical assets of the defense arsenal have the potential to apply force continuously over time, and K_I measures the flow of services associated with this potential. A second measure is the *long-run productive capacity* of military capital, K_L . Because the physical assets of the military establishment are durable, they provide benefits over time. K_L measures the value of the stream of net benefits provided over the remaining life of the defense assets.²

Estimates of military capital have typically been of the long-run productive capacity measure, K_L . However, the instantaneous productive capacity measure, K_I , is probably a more useful summary statistic because a military decisionmaker wants to determine how the durable physical assets of the defense establishment contribute to what is called "military force potential" (MF)—the value of the military output that can be produced at a particular time. The instantaneous productive capacity of military capital reflects the contribution of the durable physical assets to this measure of military capability.³

Unlike K_I , the long-run capacity measure K_L is not an input to any easily defined long-run military production function. In subsequent periods, the military capital stock may be augmented and the labor and material inputs can change. Nevertheless, K_L is a measure of military wealth and provides useful information in its own right. Not only can military wealth be compared with other types of wealth in the economy, but it might also be possible to compare the military wealth of different societies. In addition, because K_L is the measurement of capital that is normally undertaken, it is important to understand precisely what this measure means and how this information can be used to estimate K_I .

¹The U.S. military capital stock has been estimated by Musgrave (1980), and Cooper and Roll (1974). R. Shishko (1981) has estimated a military capital stock for the *Bundeswehr*.

²These two measures of capital, as well as others, are introduced by Usher (1980).

³Military force potential is introduced by Becker (1979) as "a measure based only on the national capability to apply physical force against external opponents—that is, without any reference to the external context." Hildebrandt (1980) discusses a quantitative index number to measure military force potential.

Section II develops precise analytical definitions of K_I and K_L ; the data requirements are indicated. It is shown that there are valuation and relative efficiency functions that need to be determined to estimate these two measures. Section III contains a discussion of the monetary valuation of a military asset. Both deterioration and obsolescence are considered; methods are proposed for identifying the relative efficiency functions that influence these processes. Several aggregation issues are discussed in Sec. IV. Assumptions are needed to aggregate within and across the major asset classes, and it is important to understand their significance. The effect on the military capital measures of the strategic interaction in a competitive military environment is also briefly considered in Sec. IV. Section V indicates the feasibility of determining military capital measures and the usefulness of such an undertaking to policymakers.

II. INSTANTANEOUS AND LONG-RUN CAPACITY MEASURES

The instantaneous productive capacity measure K_I identifies the monetary value of the durable assets as inputs to the "instantaneous" production of military force potential. This is the measure of the flow of services that the capital elements provide at a point in time. The long-run capacity measure K_L equals the monetary value of the net benefits provided by the durable inputs over the remainder of their service lives.

These measures of military capital are defined analytically. The definitions are carefully elaborated so that the information needed to measure military capital is clearly delineated. I introduce these two measures by examining their determination for a single class of military assets that represents some category, such as all of the tactical fighters in the defense establishment. Although only one asset class is discussed, it is convenient for the time being to suppress any asset class designation. Section IV contains a discussion of some issues that relate to aggregating the different classes of military assets—the tactical fighters, tanks, missiles, etc.—into a single measure, and the different classes of assets are identified.

INSTANTANEOUS PRODUCTIVE CAPACITY

In identifying the instantaneous productive capacity measure of military capital, I first develop a measure of the total military effectiveness provided by an asset class. This measure is the physical capital, $K_p(t)$, of the asset class. At time v , suppose that $N(v)$ units of the asset type are acquired. All units acquired in v have the same quality characteristics and are identified as the v th vintage. As these units age, the instantaneous output level they produce may decline. At time t , when each unit is age $t - v$, it produces an output that is a fraction $\Phi(t - v)$ of the output produced when new. This *deterioration function*, $\Phi(t - v)$, measures the relative efficiency of a military asset as it ages. As an example, if $\Phi(t - v)$ equals one-half, then two of the old units are militarily equivalent to one new unit of this asset when it was acquired at v . The deterioration function depends only on the age of the asset; it does not depend separately on v or t . All vintages acquired in a particular asset class are subject to the same deterioration function.

As indicated, all of the units acquired at v have the same quality (or performance) characteristics. These characteristics can, however, vary from vintage to vintage. The quality level associated with the v th vintage is designated $q(v)$. Quality level q , which can be a vector, represents the measurable performance characteristics of the equipment. For tactical fighters, this would include such characteristics as the air speed and combat range of this military asset. It is assumed that each new unit of equipment with quality level q provides $h(q)$ units of military effectiveness. The *military effectiveness function* $h(q)$ measures the relative efficiency of assets that are of different quality but of the same age. For example, if a new low quality level unit yields an $h(q)$ that is one-half that obtained from a new higher quality level unit, then two of the low quality units are militarily equivalent to one high quality unit.

The values of these functions $\Phi(t - v)$ or $h(q)$ could be obtained directly from the military decisionmaker or, alternatively, could be estimated using direct observations from the observational environment. However, either approach requires both access and additional information,

which may not be readily available. Therefore, alternative methods for estimating these functions are desirable. For an arbitrary $h(q)$ function, and for the special case in which $\Phi(t - v)$ is exponential, it is possible to identify these functions from the type of information that is likely to be available. However, decisions must be made efficiently.

Given the $\Phi(t - v)$ and $h(q)$ relative efficiency functions, the total units of military effectiveness produced by the $N(v)$ units of the v th vintage at time t can be determined. Total effectiveness of these units is $h(q)\Phi(t - v)N(v)$: Each unit acquired in v yields $h(q)N(v)$ units of military effectiveness at that time and produces a fraction $\Phi(t - v)$ of that level at t when its age is $t - v$. One can compute the measure of physical capital for the asset class by aggregating the value of military effectiveness over all vintages. If ℓ is the service life of each asset—that is, the age at which it is withdrawn from the inventory—this aggregation yields

$$K_p(t) = \int_{t-\ell}^t h(q(v))\Phi(t - v)N(v)dv. \quad (2.1)$$

To determine $K_i(t)$, the instantaneous productive capacity, a measure is needed of the monetary value of an additional unit of military effectiveness at t . This monetary value $B(t)$ is the amount the military is willing to spend for an additional unit of effectiveness at t ; Sec. III contains a discussion of how this measure of value can be identified. When $B(t)$ is multiplied by the units of physical capital $K_p(t)$, one obtains the instantaneous productive capacity measure of military capital for a particular asset class:

$$K_i(t) = B(t)K_p(t). \quad (2.2)$$

This measure of military capital is the total monetary value of all the units of military effectiveness produced at t by this type of military asset.

LONG-RUN PRODUCTIVE CAPACITY

To determine the long-run productive capacity of military capital for a particular asset class, it is necessary to identify the monetary value of the net benefits produced over the remaining life of each member. This monetary value is the amount the military is willing to spend to acquire the durable asset and is called the *demand price* of the asset. It equals the discounted value of the gross military benefits less the maintenance and repair activities that occur over the remaining life of the asset.

Gross military benefits are considered first. If the total service life of all vintages is ℓ years, then at t , each member of the v th vintage will provide military benefits for an additional $v + \ell - t$ periods until it is retired at $v + \ell$. At time s (which would be any time between t and $v + \ell$), the monetary value of a unit of military effectiveness is $B(s)$. As the age of the asset at that time is $s - v$, the deterioration function has a value of $\Phi(s - v)$. Therefore, the total military benefits provided at s by the vintage v asset would be $B(s)h[q(v)]\Phi(s - v)$. If r is the discount rate used to make the benefits of different periods commensurable, one can determine the gross military benefits of the asset $A(q, t, t - v)$, which depend on the quality level q , on t and on the age $t - v$ by computing the discounted value of the military benefits obtained over the remainder of its life:

$$A(q, t, t - v) = \int_t^{v+\ell} e^{-rs} B(s)h[q(v)]\Phi(s - v)ds. \quad (2.3)$$

As indicated, the amount that the military is willing to spend for an asset—its demand price—also depends on the level of maintenance expenditures associated with the asset. Suppose that any member of this output class with quality level q requires $m(q, s - v)$ units of maintenance and repair activity when it is $s - v$ periods old. Also, the price of each maintenance and repair unit, $R(t)$, may depend on time. Let $M(q, t, t - v)$ represent the total remaining maintenance expenditures required at t for an asset of quality level q and age $t - v$. Then the total remaining maintenance expenditure on the asset is the discounted value of all the maintenance outlays required over its remaining life:

$$M(q, t, t - v) = \int_t^{v + \ell} e^{-r(s - v)} R(s) m(q, s - v) ds. \quad (2.4)$$

As the demand price for the asset equals the gross military benefits of the assets over its remaining life minus the total remaining maintenance outlays,

$$P^d(q, t, t - v) = A(q, t, t - v) - M(q, t, t - v), \quad (2.5)$$

where $A(q, t, t - v)$ and $M(q, t, t - v)$ are determined using Eqs. (2.3) and (2.4) respectively.

The $N(v)$ units of the v th vintage have a remaining military value of $P^d(q, t, t - v)N(v)$ at t . The total remaining military value of all vintages of this equipment class is obtained by aggregating over all vintages. This yields the long-run productive capacity measure of military capital for this asset class:

$$K_L(t) = \int_{t - \ell}^t P^d[q(v), t, t - v] N(v) dv. \quad (2.6)$$

Now that $K_I(t)$ and $K_L(t)$ have been defined, it is appropriate to consider some informational aspects of the military capital determination process. Of the variables discussed, data should be available on the number of units acquired of each vintage, $N(v)$; the quality characteristics of a vintage, $q(v)$; the service life of the asset type, ℓ ; the price of repair, $R(t)$; and the level of repair, $m(q, t - v)$. However, the two relative efficiency functions, $\Phi(t - v)$ and $h(q)$, and the two valuation functions, $B(t)$ and $P^d(q, t, t - v)$, are probably not directly available. Section III shows that these four functions can be identified. The only additional information needed is the acquisition price of the asset, which may depend on the quality level selected as well as on the time of acquisition. Efficient decisionmaking by the military permits identification of the valuation and relative efficiency functions.

III. THE VALUATION AND RELATIVE EFFICIENCY OF A MILITARY ASSET

The instantaneous measure, $K_i(t)$, requires the identification of $B(t)$, the value of an additional unit of military effectiveness at t ; $\Phi(t - v)$, the deterioration function; and $h(q)$, the military effectiveness function. The long-run measure, $K_l(t)$, requires the identification of $P^d(q, t, t - v)$, the demand price at t of an asset with age $t - v$ and quality level q .

It is appropriate first to discuss some aspects of the determination of the valuation and relative efficiency functions under various information availability assumptions. One possibility is that the demand price function, $P^d(q, t, t - v)$, is known from direct observations of the prices of used military assets. If there is an active second-hand market for military equipment, this function might be estimated from the price data that is generated. Given the estimated demand price function and the maintenance expenditure function $M(q, t, t - v)$, which is defined by Eq. (2.4), it has been shown that the valuation function $B(t)$ and the relative efficiency functions $\Phi(t - v)$ and $h(q)$ can be determined.¹

Another possibility is that the deterioration function $\Phi(t - v)$ and the military effectiveness function $h(q)$ are known from other analyses. For example, deterioration might be measured using equipment availability in the operational environment. Availability is a measure of the proportion of time equipment is (on average) "available" for operational uses, and can be viewed as a relative efficiency measure that may depend on equipment age. If the failure rate of equipment or the required maintenance activities increase with the age of equipment, the availability and relative efficiency of the equipment decline with age.

One might estimate the military effectiveness function $h(q)$ by simulating the operational environment or by questioning a military decisionmaker about the relative efficiency of assets of differing quality. Given these two relative efficiency functions, the two valuation functions can be determined with the methods described in this section.

If one is fortunate enough to have direct access to the demand price function or the relative efficiency functions, the determination of the military capital stock is greatly simplified. However, both the valuation and relative efficiency functions probably need to be identified with more limited information. In this section, it is assumed that the only information available for determining these functions is the acquisition price of new military equipment, $P(q, t)$, which depends on both quality and time; the quality level q ; the maintenance expenditure function, $M(q, t, t - v)$, defined by Eq. (2.4); and the service life of the asset, ℓ . If only this information is available, it is still possible to identify the four functions as long as the number of units acquired $N(v)$, the quality level $q(v)$, and the service life of the asset ℓ are optimally selected each period. The discussion focuses on the special case of exponential deterioration. The complexity of results when exponential deterioration is not assumed is a good reason for emphasizing this special case so strongly. Straight-line deterioration is discussed in the appendix.

EXPONENTIAL DETERIORATION

Exponential deterioration occurs whenever the relative efficiency of a military asset declines at a constant rate δ as the equipment ages. This means that the percentage change in

¹Hall (1968) is probably the first to derive these functions from the price of used machines.

the value of the deterioration function decreases at the constant rate

$$-\frac{\Phi'(t-v)}{\Phi(t-v)} = \delta, \quad (3.1)$$

where δ is called the deterioration rate. Under this assumption the deterioration function must be the following form:

$$\Phi(t-v) = e^{-\delta(t-v)}, \quad t-v \geq 0. \quad (3.2)$$

Exponential deterioration implies that the relative efficiency of an asset approaches zero only as it gets very old. Although the "potential life" of an asset undergoing exponential deterioration is unlimited, the service (or economic) life of the asset may not be. The service life of an asset is the time period for which it is economical for the asset to remain in use under the specified maintenance and repair program. This service life is determined by a comparison of the incremental benefits of longer life with the incremental maintenance costs. Furthermore, one can use the optimal service life to determine the deterioration rate that is consistent with the observed service life.

Although the case of finite service life can be analyzed, there are important reasons why it is appropriate first to consider the infinite service life situation. It simplifies the analysis and, for moderately long service lives, results in a reasonable approximation of the two valuation functions.

An additional reason is that many deterioration processes converge to this pattern of total output decay. If the acquisitions of the asset are growing at a constant rate, the process of total output decay converges to a process that can be characterized by exponential (with infinite service life) deterioration for almost any arbitrary $\Phi(t-v)$ function. For example, if there is no deterioration of some asset until it is withdrawn from the inventory but the number of units acquired each period is growing at a constant rate, eventually the total level of deterioration each period will follow an exponential pattern.²

Another reason for assuming exponential deterioration with an infinite service life is that empirical studies of deterioration functions in the civilian sector of the U.S. economy have failed to reject the hypothesis that such a deterioration pattern applies.³

VALUATION AND RELATIVE EFFICIENCY WITH INFINITE SERVICE LIFE

An examination of Eqs. (2.3) and (2.5) indicates that the demand price of an asset $P^d(q, t, t-v)$ depends on $B(s)$, the value of an additional unit of military effectiveness at s . Therefore, it is appropriate to determine the instantaneous value measure before the value of the asset itself.

When the military acquires an additional unit of a physical asset of quality level q , it incurs a resource cost of $P^s(q, t)$. This cost, called the supply price of the asset, may vary with time because of technological change in the industry producing it; as time passes it may be cheaper

²The convergence result is discussed by Feldstein and Rothschild (1974). The situation in which deterioration of the asset does not occur until it is withdrawn from the inventory, called one-hoss shay deterioration, is discussed in the appendix.

³See Coen (1980) for a discussion of these empirical results.

to produce a given quality level. The supply price, however, is assumed not to vary with the number of units acquired each period; the marginal cost of the asset is constant at any time.

Throughout its service life, the asset directly contributes to military capability and provides *gross* military benefits. At t , the discounted value of these gross military benefits is computed with Eq. (2.3) for the special case in which the asset is new. As indicated by Eq. (2.5), to obtain the *net* benefits, or demand price of the asset, $P^d(q,t)$, it is necessary to deduct the discounted value of maintenance and repair expenditures $M(q,t)$ incurred over the life of this new asset. These expenditures are computed with Eq. (2.4) for this special case in which $t = v$. When the asset is new, it is convenient to suppress the age argument of the demand price function as well as the maintenance requirements function.

As more units are acquired each period, the demand price falls because of diminishing marginal military value.⁴ At the efficient solution, the demand price is equal to the constant supply price:

$$P^d(q,t) = P^s(q,t) = P(q,t), \quad (3.3)$$

where $P(q,t)$ is the common value of the demand and supply price. This value is the acquisition price of the asset. At every t , the acquisition price of the asset equals the discounted value of the net benefits provided. For the situation in which the service life is infinite,

$$P[q(t),t] = \int_t^{\infty} B(s)h[q(t)]e^{-(r+\delta)(s-t)}ds - M[q(t),t]. \quad (3.4)$$

Totally differentiating this expression with respect to t yields an expression for the value of military benefits obtained at t from the last unit acquired:⁵

$$B(t)h[q(t)] = (P + M)(r + \delta + \dot{h}/h) - (\dot{P} + \dot{M}) \quad (3.5)$$

where the dot ($\dot{}$) over a function indicates the *total* derivative of the function with respect to time.

The expression on the left-hand side equals the value of an additional unit of the asset of t . It is the product of the units of effectiveness provided $h[q(t)]$ and the monetary value of an additional unit of effectiveness $B(t)$. The expression on the right-hand side of the equality is called the user cost, $C(t)$, of an additional unit of the new asset at t . By computing the user cost, one determines the value of an additional unit of the military equipment at a point in time.

To interpret Eq. (3.5), it is helpful to define the full cost of a new asset, $\pi(q,t)$, as the sum of the acquisition cost, $P(q,t)$, and the maintenance cost, $M(q,t)$:

$$\pi(q,t) = P(q,t) + M(q,t). \quad (3.6)$$

When an additional unit is acquired at t , the total value of the resources committed to this unit equals the full cost.

With the notion of full cost, the user cost expression can be written

$$C(t) = \pi r + \pi \delta + \pi \dot{h}/h - \dot{\pi}. \quad (3.7)$$

⁴There is diminishing marginal value to increases in physical capital, $K_p(t)$, during a particular period. It simplifies the discussion to suppress this argument of the demand price function.

⁵This derivation has similarities to the approach taken by Hall (1968), although he doesn't distinguish between acquisition price and maintenance costs.

Each of the terms on the right-hand side of the equality represents a particular type of cost associated with acquiring an additional unit at t rather than postponing its acquisition one period. The first term, πr , equals the opportunity cost of acquiring one more unit at t rather than one period later. A cost is incurred because a return elsewhere in the economy of πr is being forgone.

The second term, $\pi \delta$, is the cost associated with deterioration. If a unit is acquired in period t , then beginning next period it will yield remaining gross benefits that are δ percent lower than those obtained from a unit acquired the next period. Postponing the purchase of the marginal unit one period avoids this decline in output and the savings equals $\pi \delta$.

To interpret the third term, suppose that quality is rising over time. Then, there is a cost $\pi \dot{h}/h$ associated with buying the lower quality unit at t rather than the higher quality unit one period later. This is the cost associated with technological obsolescence.

To interpret the fourth term, suppose that π is declining over time. The cost associated with acquiring the last unit at t rather than waiting for a lower price one period later equals $-\dot{\pi}$.

Equation (3.7) applies whether or not the quality level is selected optimally each period. When the quality level of the asset can be adjusted each period, its level should be chosen so as to minimize the full cost of an asset per unit of military effectiveness.

Additional justification for choosing the optimal quality level in this fashion is found below under *Research and Development*. The efficiency condition obtained by solving this problem is the same as that obtained by solving the more general dynamic optimization problem.⁶ The problem at each t is to choose q to minimize $\pi(q, t)/h(q)$. By differentiating this cost per unit of effectiveness with respect to q , one obtains

$$h_q/h = \pi_q/\pi, \quad (3.8)$$

where π_q and h_q represent the derivatives of π and h with respect to q . This condition states that the quality level should be selected each period so that the percentage change in military effectiveness equals the percentage change in the full cost of the asset. Satisfying this condition defines an optimal π/h function that varies over time only because of technical change, and not because of changes in the quality level selected. This suggests that there is no technological obsolescence when quality is a choice variable that can be seen directly by rewriting the last two terms of Eq. (3.7) in an equivalent form.

$$\pi \dot{h}/h - \dot{\pi} = \pi h_q/h(\dot{q}) - \pi_q \dot{q} - \pi_t, \quad (3.9)$$

where π_t represents the partial derivative of π with respect to t . Combining the right-hand side of Eq. (3.9) with Eq. (3.8), which is the condition that applies for the selection of the optimal quality level, yields

$$\pi \dot{h}/h - \dot{\pi} = -\pi_t. \quad (3.10)$$

Therefore, under optimal quality selection Eq. (3.7) can be rewritten

$$C(t) = \pi r + \pi \delta - \pi_t, \quad (3.11)$$

and there is no obsolescence cost when q is optimally selected. There is no change each period in full cost per unit of effectiveness from changes in quality alone. The term π_t remains part

⁶See the discussion associated with Eq. (3.33) and footnote 14.

of the user cost expression. This term can be defined as the capital loss associated with a technologically induced change in the full cost. If π_t is negative, there is a cost associated with buying a unit at t at a higher full price than is available the subsequent period.

It is appropriate to restate the equality between the incremental benefit and the user cost that applies at t when both the number of units acquired and the quality level are optimally selected. When stated in terms of the benefit and cost of an additional unit of military effectiveness, the following condition applies:

$$B(t) = \pi/h(r + \delta) - \pi_t/h. \quad (3.12)$$

To determine $B(t)$ it remains necessary to identify the $h(q)$ function. Fortunately, this function is readily identifiable whenever the optimal quality level is selected and Eq. (3.8) applies. This condition determines the optimal relationship between time and quality, $t(q)$. This Eq. (3.8) may be rewritten in terms of the logarithmic derivatives:

$$\frac{d \ln h(q)}{dq} = \frac{\partial \ln \pi[q, t(q)]}{\partial q}. \quad (3.13)$$

As the military effectiveness function is a relative efficiency function, its value at some base quality level q_0 can be set equal to an arbitrary value. It is convenient to set $h(q_0)$ equal to $\pi[q_0, t(q_0)]$. Then one can use Eq. (3.13) to integrate from $\ln h(q_0)$ to $\ln h(q)$, obtaining

$$\ln h(q) = \ln \pi[q_0, t(q_0)] + \int_{q_0}^q (\partial \ln \pi / (\partial q)) [\xi, t(\xi)] d\xi. \quad (3.14)$$

The second term on the right-hand side of Eq. (3.14) can be interpreted as the aggregation of the changes in full cost attributed to changes in the selected quality level that have occurred each period. This term *does not* include the changes in cost attributed to exogenous factors, and it is not legitimate to equate $\ln h(q)$ and $\ln \pi[q, t(q)]$. This point is clarified if one recognizes that

$$\begin{aligned} \ln \pi[q, t(q)] &= \ln \pi[q_0, t(q_0)] + \int_{q_0}^q (\partial \ln \pi / (\partial q)) [\xi, t(\xi)] d\xi \\ &\quad + \int_{q_0}^q (\partial \ln \pi / (\partial t)) [\xi, t(\xi)] t'(\xi) d\xi. \end{aligned} \quad (3.15)$$

The last term on the right-hand side of Eq. (3.15) equals the aggregation of the percentage changes in price, $(\partial \ln \pi / (\partial t)) = \pi_t/\pi$, that have occurred each period. This is true because

$$\int_{q_0}^q (\pi_t/\pi) t' d\xi = \int_{t_0}^t (\pi_t/\pi) d\tau, \quad (3.16)$$

where t and t_0 are the time periods at which q and q_0 are selected. The military effectiveness $h(q)$ can be identified as long as the price functions $\pi(q, t)$ and $t(q)$ are known. The latter function results from the selection of the optimal quality level each period. To identify $h(q)$, there is no need to simulate the operational environment or make direct inquiries of a decisionmaker.

Turn now to $P^d(q, t, t - v)$, the demand and price at t of an asset of quality level q , and age $t - v$. Exponential deterioration with infinite service life is still assumed. When the military asset acquired at v is age $t - v$, it yields a military output that is a proportion $e^{-\delta(t-v)}$ of the output yielded when it was new. With Eqs. (2.3) and (2.5), the demand price at t for this asset can be written

$$P^d[q(v), t, t - v] = \int_t^\infty e^{-\delta(t-v)} B(s) h[q(v)] e^{-(\delta + r)(s-t)} ds - M[q(v), t, t - v], \quad (3.17)$$

where the required maintenance costs are determined with Eq. (2.4). $B(s)$ and $h(q)$ are both identifiable with available data and are determined by Eqs. (3.12) and (3.14) respectively. Although it is difficult to evaluate Eq. (3.17), it can be shown that⁷

$$P^d[q(v), t, t - v] = \frac{h[q(v)]}{h[q(t)]} P[q(t), t] e^{-\delta(t-v)} + \frac{h[q(v)]}{h[q(t)]} M[q(t), t] e^{-\delta(t-v)} - M[q(v), t, t - v]. \quad (3.18)$$

The first two cost terms are multiplied by $\{h[q(v)]/h[q(t)]\}e^{-\delta(t-v)}$, the efficiency of the vintage v unit relative to the vintage t unit. Therefore, the first term equals the price of new vintage t equipment times the relative efficiency of the old vintage v equipment. The last two cost terms equal the total maintenance expenditure for a portion of new equipment equal to the relative efficiency of the old equipment minus its remaining maintenance requirements.

A simplification of Eq. (3.18) occurs if there is no change in quality over time, and the maintenance requirements equal a proportion, $e^{-\delta(t-v)}$, of the new equipment requirements when the asset age is $t - v$. The maintenance requirements each period would then decline at the same rate δ as the asset is deteriorating. An asset of age $t - v$ would require a level of maintenance activity equal to $me^{-\delta(t-v)}$, and the total maintenance expenditure requirements for the remaining life of the asset would be $M(t)e^{-\delta(t-v)}$. These assumptions imply that demand price is computed as

$$P^d(t, t - v) = P(t) e^{-\delta(t-v)}. \quad (3.19)$$

It equals the cost of buying the proportion of new equipment that achieves the relative efficiency of the used equipment.

These special assumptions also permit simplifying the user cost expression. Suppose that the price of maintenance activity R is a proportion σ of the acquisition price P , and that both of these costs are declining at a rate γ each period. With Eq. (3.7) the user cost expression can be written

⁷Hall (1968) has derived an equation similar to Eq. (3.18).

$$C(t) = P(t)(r + \delta + \sigma m + \gamma). \quad (3.20)$$

This special case also yields a simple relationship between $K_I(t)$ and $K_L(t)$, the instantaneous and long-run productive capacity measures of military capital for a single asset. In this special case, it can be shown that

$$K_I(t) = (r + \delta + \sigma m + \gamma)K_L(t). \quad (3.21)$$

For this particular class of military assets, the instantaneous measure of capital is a proportion of the long-run measure. Although this result greatly simplifies the analysis, one must evaluate the validity of the underlying assumptions before using this simple relationship.

DEMAND PRICE AND USER COST WITH A GENERAL DETERIORATION FUNCTION

Following is a brief discussion of the demand price and user cost functions that apply when a general deterioration function, $\Phi(t - v)$, applies and an infinite service life is assumed. Equation (3.18) can be used as a point of reference to help understand the general demand price function. The discussion of this general function will, however, be only suggestive; it is quite difficult to derive the demand price function for the general case.⁸

The constancy of the deterioration rate δ greatly simplifies the determination of $P^d(q, t, t - v)$. This constancy implies that there is no economic difference between an old asset and a fraction of a new asset equal to the relative efficiency of the old asset. From the standpoint of the *total replacements* required to maintain a particular capability level, the two assets are equivalent under exponential deterioration: Both will experience the same rate of decline in output and will need to be replaced at the same rate. However, if the deterioration rate is not constant, there is an important difference between a fraction of a new asset and an old asset whose relative efficiency equals that fraction. For example, suppose an old asset requires more total replacement expenditures (including the replacements of the replacements, etc.) than does the relative efficiency fraction of the new asset. This additional replacement outlay will tend to reduce the demand price to a level below that based on Eq. (3.18).

Suppose we identify the total discounted value at t of the replacement expenditure of an asset of quality level q and age $t - v$ as $Y(q, t, t - v)$. When the asset is new, the age argument is suppressed and the replacement expenditure equals $Y(q, t)$. It can be shown that the replacement expenditure function can be obtained directly from the deterioration function, $\Phi(t - v)$. The replacement expenditure function affects the demand price of an asset in the same manner as the maintenance expenditure function. For a general deterioration function $\Phi(t - v)$, the following extension of Eq. (3.18) can be obtained:

$$\begin{aligned} P^d[q(v), t, t - v] = & \frac{\Phi(t - v)h[q(v)]}{h[q(t)]} \{ [P[q(t), t] + M[q(t), t] \\ & + Y[q(t), t] \} - M[q(v), t, t - v] \\ & - Y[q(v), t, t - v]. \end{aligned} \quad (3.22)$$

The additional terms not included in Eq. (3.18) are

⁸For a derivation of demand price when there is a general deterioration function; see Hall (1968).

$$\frac{\Phi(t-v)h[q(v)]}{h[q(t)]} Y[q(t),t] - Y[q(v),t,t-v].$$

These two terms equal a fraction of the total discounted replacement expenditures required for new quality equipment $q(t)$ minus the replacement expenditures associated with the old quality equipment $q(v)$, where the fraction equals the efficiency of the old equipment relative to the new. This difference equals zero if and only if exponential deterioration is assumed.

The user cost expression, $C(t)$, for a general deterioration function would be the same as Eq. (3.7) with the exception that the *deterioration cost* term $\pi\delta$ would be replaced by a more general term that reflects the *economic depreciation* of the new asset at t . Economic depreciation is the decline in the value of the asset as it ages. Moving the maintenance requirement function, $M[q(v),t,t-v]$, to the left side of Eq. (3.22) yields the full demand price of the asset, $\pi[q(v),t,t-v]$.⁹ Economic depreciation can be defined as the derivative of this function with respect to the age $t-v$. In the determination of $C(t)$, the relevant economic depreciation would be computed with the full price of the *new* asset $\pi(q,t)$. The economic depreciation component of user cost would be $\partial\pi(q,t)/\partial(t-v)$. This economic depreciation cost depends on all deterioration rates and associated replacements.

If the deterioration rate is a constant δ , the replacement requirement at every t will always be a fixed proportion of the remaining quantity of the asset as the asset ages. This is why $\pi\delta$ is the correct measure of economic depreciation in Eq. (3.7). However, when the deterioration rate is not constant, it is necessary to know the full demand price function $\pi(q,t,t-v)$ to determine $\partial\pi(q,t)/\partial(t-v)$. To obtain $\pi(q,t,t-v)$ one needs to compute Eq. (3.22), the general demand price expression. The complexity of this equation is one reason for investigating the appropriateness of assuming exponential deterioration.

FINITE SERVICE LIFE

As already indicated, it is appropriate to retire an asset when the benefits from additional service life equal the additional maintenance cost incurred. To analyze the finite service life situation, suppose that the level of maintenance activity increases each period during the asset's life at the rate θ . Then, for an asset of age $t-v$, the maintenance requirement would be $me^{\theta(t-v)}$. If the price of a unit of maintenance activity is a constant R , maintenance expenditure for new equipment, M , would be the discounted value of the expenditures that take place over the service life ℓ of the asset.

$$M = Rm \int_t^{t+\ell} e^{(r-\theta)(s-t)} ds = Rm \frac{1 - e^{(r-\theta)\ell}}{r - \theta}. \quad (3.23)$$

To simplify the analysis without affecting the conclusions, quality is also assumed to be unchanging over time so that the $h(q)$ function can be ignored. It is also simplest to assume

⁹One could define a long-run productive capacity measure of capital using the full demand price to weight the capital elements. However, there seems to be a long-standing tradition in capital stock estimation to weight the capital elements using the acquisition price for new assets, and some "adjusted acquisition price" for deteriorated assets. Using the demand price of the asset, $P^d(q,t,t-v)$, to weight the different assets is probably a continuation of this estimation tradition.

that acquisition price P of the asset is unchanging over time. Constant acquisition price combined with a constant maintenance price R is consistent with the value of gross military benefits B at any time remaining constant. It is assumed that no other factors (such as a change in the external environment) cause B to vary. A decisionmaker would equate this acquisition price to the discounted value of the net military benefits produced by an additional unit of the asset over its service life:

$$P = B \int_t^{t+\ell} e^{-(r+\delta)s} ds - Rm \frac{1 - e^{-(r+\delta)\ell}}{r + \delta} \quad (3.24)$$

By evaluating the integral in this equation, one obtains the condition equating the value of the instantaneous services provided by an additional unit and the user cost C :

$$B = \frac{(P + M)(r + \delta)}{1 - e^{-(r+\delta)\ell}} \equiv C. \quad (3.25)$$

When this expression is compared with Eq. (3.5), it is seen that user costs are being scaled up by the factor $[1 - e^{-(r+\delta)\ell}]^{-1}$; with the service life now finite, the imputed cost each period must be greater.¹⁰ For a reasonably large service life, it might be appropriate to ignore this scaling factor to simplify the analysis.

The optimal service life is determined by setting the derivative of Eq. (3.24) with respect to ℓ equal to zero.¹¹

$$Be^{\delta\ell} = Rme^{\theta\ell}. \quad (3.26)$$

The term on the left is the value of an additional period of service life of an asset that has depreciated at a rate δ for ℓ periods. The term on the right equals the maintenance cost of additional service life. It is appropriate that ℓ should be selected so that these two terms are equal.

After one substitutes for B using Eq. (3.25), Eq. (3.26) is written

$$\frac{(P + M)(r + \delta)}{1 - e^{-(r+\delta)\ell}} e^{\delta\ell} = Rme^{\theta\ell}. \quad (3.27)$$

Because this expression applies whenever ℓ is selected optimally, it can be used to determine the depreciation rate δ : It is the rate that leads to ℓ being selected, given the values of the other known parameters. Suppose $\hat{\delta}$ satisfies Eq. (3.27). Then the applicable deterioration functions would be $\Phi(t - v) = e^{\hat{\delta}(t-v)}$. Thus the last of the four functions is identifiable. The two valuation functions $B(t)$ and $P^d(q, t, t - v)$ and the two relative efficiency functions $h(q)$ and $\Phi(t - v) = e^{\hat{\delta}(t-v)}$ can be identified when the specified information is available. The deterioration function Φ is identified only for the exponential case in this analysis. Although the other

¹⁰An equivalent form of the user cost expression is

$$C = (P + M)r + [(P + M)(r + \delta)e^{-(r+\delta)\ell}]/[1 - e^{-(r+\delta)\ell}].$$

One can view $(r + \delta)e^{-(r+\delta)\ell}/[1 - e^{-(r+\delta)\ell}]$ as the exponential depreciation rate $\bar{\delta}$ for infinite service life that leads to a user cost level equal to that of Eq. (3.23). The depreciation rate $\bar{\delta}$ is discussed in the appendix.

¹¹Alternatively, Eq. (3.24) can be obtained with the dynamic optimization technique used in the subsection dealing with Research and Development. This optimization technique can also be used to determine the optimal value of the maintenance parameter m , if this parameter influences the deterioration rate δ .

functions are identified without imposing the same type of restrictions, there are as indicated several reasons for assuming that deterioration is exponential.

RESEARCH AND DEVELOPMENT

Do any of the results of this section change when R&D expenditures are introduced? This class of expenditures can affect an acquisition process in different ways, and two possibilities are evaluated: First cost-reducing expenditures are considered, then product improvement R&D. For simplicity, infinite service life is assumed.

Suppose that at the beginning of the planning horizon, a research and development outlay of D is expended. These expenditures are designed to influence the acquisition price function that applies each period, so that one would have $P(q,t,D)$. Expenditure D is an additional choice variable during the acquisition process, and its optimal value \bar{D} determines the function $P(q,t,\bar{D})$ that applies each period. As \bar{D} does not change from period to period during the acquisition process, it can be subsumed into the functional form of the acquisition price function; and all of the results that have been discussed continue to apply.

To see why R&D expenditures can be ignored in this situation, consider a special case. Suppose that the acquisition price of an asset is "infinitely" high unless a specified level of R&D is expended. This level of expenditure is like a set-up charge that needs to be expended if any units are to be acquired. The cost of an additional unit does not vary with the number acquired. Such a set-up charge would not be part of either the user cost or the demand price of an additional unit and would have no direct bearing on the computation of either measure of military value.

A somewhat different situation occurs when a modernization expenditure Z is incurred each period in order to affect the rate of quality change \dot{q} ; this is the product improvement R&D situation. Given the level of these expenditures, the rate of change of quality also depends on the quality level q from which one is modernizing. The functional relationship between the rate of change of quality and Z and q is represented by

$$\dot{q} = F(Z,q). \quad (3.28)$$

The expenditure Z does not vary with the number of units acquired.

It seems plausible for the acquisition price to depend on the rate of quality change that occurs. Presumably, it would be cheaper to acquire an asset when the change in quality is small than it would be when there is a substantial change in quality. Therefore, the acquisition price function can be expressed $P(q,t,\dot{q})$; the direct dependence of this function on the quality level and on time is retained.¹²

To develop the analysis further, it is convenient to use the measure of physical capital $K_p(t)$ defined by Eq. (2.1). This measure equals the total level of military effectiveness produced by all the units of this asset class at t . Under the infinite service life assumption, $K_p(t)$ has a time rate of change equal to

$$\dot{K}_p(t) = h[q(t)]N(t) - \delta K_p(t). \quad (3.29)$$

The change in physical capital at t equals the total level of effectiveness embodied in the new

¹²Robert Shishko has proposed a variation of the R&D model in which the maximum quality level is a choice variable. He also helped to clarify the distinction between cost-reducing and product improvement R&D. There is currently no consensus concerning how R&D should be modeled. This lack of consensus supports the overall conclusion of this subsection that R&D expenditures should not be considered when one is estimating military capital.

equipment minus the proportion of existing physical capital that deteriorated during the period.

The problem facing the decisionmaker is to select the number of units acquired $N(t)$ and the level of modernization expenditure $Z(t)$ at every t in order to optimally determine the time path of both the physical capital stock and the quality level. If this stock has a gross monetary value of $V[K_p(t)]$, the marginal value of an additional unit of physical capital $V'[K_p(t)]$ equals $B(t)$, the monetary value of an additional unit of military effectiveness. The following Hamiltonian applies to this problem:

$$H = e^{-\rho t} \{ V(K_p) - P(q, t, F(Z, q))N - Z + Y^k[h(q)N - \delta K_p] + Y^q F(Z, q) \} \quad (3.30)$$

where Y^k and Y^q are costate variables indicating respectively the value of additional capital and additional quality. The function F , defined by Eq. (3.28), has been substituted for \dot{q} in the acquisition price function.

When this expression is maximized with respect to N and Z , and the rates of change in the costate variables are properly determined, one obtains

$$V'[K_p(t)] \equiv B(t) = \frac{P}{h}(r + \delta + \dot{h}/h) - \frac{\dot{P}}{h}, \quad (3.31)$$

where \dot{P} is the total derivative of acquisition price with respect to time. This is the same form of the equality between marginal benefits and user costs as indicated in Eq. (3.5).¹³ However, the total derivative of price with respect to time \dot{P} in Eq. (3.31) is a complex expression:

$$\dot{P} = P_q \dot{q} + P_t + P_F F_Z \dot{Z} + P_F F_q \dot{q}, \quad (3.32)$$

where the subscript indicates the argument of the function with respect to which the derivative is being taken. From Eq. (3.12) when optimal quality choice without modernization expenditures was considered, only the partial derivative of price with respect to time was included in the user cost expression. The indirect effects on acquisition price of the time rates of change in variables, which are included in Eq. (3.31), are not relevant unless modernization expenditures are incurred.

With regard to the optimal quality selection situation discussed earlier, the major complication encountered is that the identification of $h(q)$ is far more complex. Equation (3.8) no longer applies when expenditure Z can be incurred each period. The percentage change in military effectiveness does not equal the percentage change in acquisition price. Instead, for this type of R&D situation,

$$Y^k = P/h = P_q/h_q + \frac{rP_F N + r/F_Z - F_q/F_Z - P_F \dot{N} - N \dot{P}_F + \dot{F}_Z/(F_Z)^2}{h_q N} \quad (3.33)$$

This equation specializes to Eq. (3.8) whenever there is no product improvement R&D.¹⁴ Therefore, this analysis also demonstrates the validity of minimizing the acquisition price per unit of military effectiveness when choosing the quality each period in the earlier discussion.

Although Eq. (3.33) can be used to identify $h(q)$, both the uncertainty associated with the F function as well as the analytical complexity associated with the determination of h probably make it inappropriate to undertake the identification of $h(q)$ using Eq. (3.33).

¹³The only difference results from the fact that maintenance and repair expenditures are being ignored.

¹⁴If there is no possibility for product improvement, P_Z, F_Z, \dot{F}_Z , and $F_q = 0$. Therefore, with the quality level a choice variable one would have $Y^k = P/h = P_q/h_q$. With the exception that maintenance expenditures are being ignored, one obtains a result equivalent to Eq. (3.8).

For actual computations of the military capital stock, it is probably wise to ignore R&D expenditures. When they occur at the beginning of the planning horizon, they should be ignored. When modernization expenditures can occur each period, the identification of $h(q)$ is greatly complicated. Although empirical analysis is probably appropriate to determine which model of R&D is more relevant, it is probably best to ignore these expenditures in military capital stock analysis at the present time.

IV. AGGREGATION AND INTERACTION OF MILITARY ASSETS

This report has shown how the services provided at some time by a single class of military assets, such as tactical fighters, can be valued in monetary terms. The value of any asset in the class at that time can also be determined. However, we wish to obtain a single monetary measure of all the durable defense assets in the military arsenal including all the equipment, facilities, and inventories. We are also interested in intertemporal commensurability; it is desirable that the same yardstick be used over time so that one period's level of military capital can be compared with the level of some other period.

First consider the determination of the aggregate measure of productive capacity, $K_i(t)$. Although the intra-asset aggregation procedure described in Sec. III continues to apply, it is appropriate at this point to develop this procedure from a more fundamental position.

INTRA-ASSET AGGREGATION

Each asset class is designated by the index $i = 1, \dots, I$. Within a particular class $N^i(v)$ units with quality level $q^i(v)$ are acquired in year v . The applicable deterioration function is $\Phi^i(t - v)$. At t , the $N^i(v)$ units of vintage v are combined with the labor $L^i(t, v)$ and the material $G^i(t, v)$ assigned to it to yield military output $Q^i(t, v)$ in accordance with the function

$$Q^i(t, v) = f^i\{w^i[q^i(v), N^i(v) \Phi^i(t - v)], L^i(t, v), G^i(t, v)\} \quad i = 1, \dots, I, \quad (4.1)$$

where the function w^i reflects the fact that quality $q^i(v)$ is embodied in the vintage v equipment. However, as long as the production function is homogeneous in $N^i(v)$, $L^i(t, v)$ and $G^i(t, v)$, it is true that¹

$$w^i = h^i[q^i(v)] N^i(v) \Phi^i(t - v), \quad i = 1, \dots, I. \quad (4.2)$$

This equation states that under the homogeneity assumption, there is a military effectiveness function $h^i[q^i(v)]$ that determines the relative efficiency of military assets of different quality. This result permits Eq. (4.1) to be rewritten as

$$Q^i(t, v) = f^i\{h^i[q^i(v)] N^i(v) \Phi^i(t - v), L^i(t, v), G^i(t, v)\}, \quad i = 1, \dots, I. \quad (4.3)$$

At t , this type of production function applies for every vintage of the i th class. The problem facing the military decisionmaker is to optimally allocate to the different vintages the total labor, $L^i(t)$, and material, $G^i(t)$, that are available for the i th asset class. If the total output produced by the i th class $Q^i(t)$ is the sum of each vintage's output, then the problem is to select $L^i(t, v)$ and $G^i(t, v)$ to

$$\text{Max } Q^i(t) = \int_{t-\ell^i}^t Q^i(t, v) dv, \quad (4.4)$$

¹This result has been proved by Willig (1976).

subject to

$$\int_{t-\ell^i}^t L^i(t,v)dv = L^i(t)$$

$$\int_{t-\ell^i}^t G^i(t,v)dv = G^i(t),$$

where each $Q^i(t,v)$ is determined by Eq. (4.3) and ℓ^i is the service life of the i th asset class.

When this problem is solved, an aggregate production function emerges that the decision-maker can be viewed as facing. The arguments of this function are the aggregate physical capital $K_p^i(t)$, aggregate labor $L^i(t)$, and the aggregate material $G^i(t)$ for the i th class.²

$$Q^i(t) = Q^i[K_p^i(t), L^i(t), G^i(t)], \quad i = 1, \dots, I, \quad (4.5)$$

where physical capital equals the units of military effectiveness produced by the i th class and is defined by Eq. (2.1).

Total military output, called military force potential MF, depends on all of the output levels produced by the I asset classes of the defense establishment. The functional relationship is written

$$MF(t) = f[Q^1(t), \dots, Q^i(t), \dots, Q^I(t)], \quad (4.6)$$

where the output obtained from each class is determined by Eq. (4.5).

INTER-ASSET AGGREGATION

Given a production function of the general form indicated by Eq. (4.6), the problem is determining when aggregate military capital can be written as a separate argument of this function. If certain restrictions on the functional form of Eq. (4.6) are satisfied, then a capital aggregate can be formed. Unfortunately, such functional form restrictions are unlikely to be satisfied in practice.³ However, as long as the relative prices of the different types of physical capital do not change, a measure of aggregate capital can still be formed. The price of an effectiveness unit of each type of physical capital $K_p^i(t)$ is its user cost $C_p^i(t)$, which is obtained by dividing the user cost per unit, $C^i(t)$, as determined by Eq. (3.11), by $h(q)$, the military effectiveness per unit. Therefore, if one assumes that the user cost of each physical capital type is changing at the same rate $-\gamma$, it is possible to aggregate the various measures of physical

²For a discussion of this theorem, see Fisher (1969).

³For example, if there are two physical capital types, K_p^1 and K_p^2 , the marginal rate of substitution of K_p^1 for K_p^2 must be independent of the labor and material utilized for a capital aggregate to be formed within the production function itself. The aggregate production function would necessarily be of the form $MF = f(J(K_p^1, K_p^2), L, G)$ for the independence property to be obtained. Leontief (1947) was the first to prove this aggregation theorem. As Fisher (1969) has shown, this aggregation theorem applies to intra-asset aggregation as developed in Sec. IV. It would also apply to inter-asset aggregation if the production functions for each asset class were all of the form

$$Q^i = f[g(i, K_p^i), L^i, G^i], \quad i = 1, \dots, I,$$

and $MF = \Sigma Q^i$. This production function implies that one military production function applies to all military assets, and all differences in the I asset classes are embodied in the physical capital. It is not reasonable to assume that the same production function applies to both strategic and conventional weapons.

capital.⁴ Because the user costs $C_p^i(t)$ equal the monetary value of an additional unit of the asset $B^i(t)$, these costs, measured at some base period t_0 , are the appropriate weighting factor. If these base period costs equal $C_p^i(t_0)$, then at t the aggregate measure of instantaneous productive capacity, $K_I(t; t_0)$, would be

$$K_I(t; t_0) = \sum_{i=1}^I C_p^i(t_0) K_p^i(t) = \sum_{i=1}^I K_i^i(t; t_0), \quad (4.7)$$

where the weighted sum of the levels of physical capital equals the simple sum of the instantaneous measures computed with the base prices. The user cost, $C_p^i(t_0)$, would be determined by dividing $C^i(t_0)$ by the level of military effectiveness in the base period, $h^i(q(t_0))$. However, the level of military effectiveness in the base period $h^i(q(t_0))$ can be equated to the base period price $\pi^i[q^i(t_0), t_0]$ whenever quality is selected optimally (see p. 10). Therefore, for the base period, the ratio π/h would equal unity. If one assumes that the percentage change in full price each period is a constant such that $\pi_t^i = -\gamma^i \pi^i$, the relevant user cost would be⁵

$$C_p^i(t_0) = r + \delta^i + \gamma^i. \quad (4.8)$$

The influence of differences in the quality of different vintages would still be felt because Eq. (3.14) would be used to determine the military effectiveness function $h^i(q)$, which is a key ingredient of K_p^i .

The long-run productive capacity measure of military capital $K_L(t)$ does not enter as an argument of an instantaneous production function such as Eq. (4.6). It measures the military value of the existing equipment over its remaining life. For measurements of K_L to be comparable over time, it is necessary that the demand price, computed using Eq. (3.18), be determined using the acquisition prices that apply during the base period. For the i th asset class,

$$K_L^i(t; t_0) = \int_{t-Q^i}^t P^i d[q^i(v), t_0, t - v] N^i(v) dv, \quad i = 1, \dots, I, \quad (4.9)$$

and the aggregate long-run productive capacity measure $K_L(t; t_0)$ would be the sum of the measures for each asset class:

$$K_L(t; t_0) = \sum_{i=1}^I K_L^i(t; t_0). \quad (4.10)$$

A simple summation is legitimate because each additional dollar of asset value represents the same contribution to military capability over the remaining lives of the different assets.

Is there ever a simple relationship between $K_L(t; t_0)$ and $K_I(t; t_0)$? For the special case in which the quality within each asset class is unchanging, the deterioration is exponential, and the maintenance requirements are declining correspondingly, Eq. (3.21) would be satisfied and

⁴This result is called the Hicks commodity aggregation theorem (1946). This theorem has been applied to capital aggregation by Brown and Chang (1974). If the full price π^i is varying at a constant rate, the same quality level would be selected each period. Nevertheless, it is probably best to make this constant rate assumption when computing aggregate capital.

⁵Although the capital loss factor γ is indicated as being the same across assets, in an actual situation this assumption would be an approximation. Therefore, each asset-specific γ^i is used in Eq. (4.8), even though it is necessary to assume that each π^i is changing at the same rate to aggregate the physical capital elements.

one can compute aggregate instantaneous capital as

$$K_I(t; t_0) = \sum_{i=1}^I (r + \delta^i + \sigma^i m^i + \gamma^i) K_L^i(t; t_0). \quad (4.11)$$

Even for this special case, the instantaneous productive measure will not normally be a constant proportion of the long-run productive measure over time. The only useful situation in which this constant proportion occurs is when each $K_L^i(t; t_0)$ remains a fixed proportion of aggregate $K_I(t; t_0)$ over time. The long-run productive measure for each asset class must be growing at the same rate for aggregate instantaneous capital to be intertemporally proportional to aggregate long-run military capital.

STRATEGIC INTERACTION

Thus far I have not considered the effects of the actions of an adversary on the value of military capital. As time passes and an adversary introduces new equipment, military capital probably experiences what can be called *competitive obsolescence*.

One approach to incorporating competitive obsolescence in military capital determination assumes that the decline in performance resulting from the actions of an adversary occurs at a constant rate throughout the life of the asset. Competitive obsolescence is then handled much like exponential deterioration as discussed in Sec. III. A gross-deterioration rate can be defined that includes both physical deterioration and competitive obsolescence; this rate could be estimated with Eq. (3.27).

The problem with this approach is that competitive obsolescence is reduced to a process that is extremely mechanistic; the effects of specific actions taken by an adversary are largely ignored.

Actually, the hypotheses that can be formed concerning an adversary's actions and reactions are quite varied and depend in part on the degree of sophisticated behavior.⁶ For example, suppose that an adversary is believed to be *ultra-naive* because he does not adjust his military asset levels in response to one's own acquisitions. It is then legitimate to view the adversary's capital as being invariant with respect to one's own stock when determining military capital.

However, it is probably *naive* to believe that the adversary does not make any adjustment. If the adversary does base his asset selection on one's own capability level, then the *sophisticated* decisionmaker would take account of the adversary's reaction function when selecting the level of military capital. The demand price for an additional unit would take account of the adversary's reaction; no separate adjustment is needed for competitive obsolescence.

Super-sophisticated behavior is the next degree of sophistication. This behavior type occurs when one knows that an adversary is reacting to one's own reaction function when selecting his level of capital. It is appropriate in this case to react to his reaction to one's own reaction function by signaling to the adversary the reaction function that it is in one's interest to have the adversary believe applies.

Moving up the sophistication chain in this fashion suggests that the subtlety and diversity of the interaction patterns may outpace one's ability to identify the interaction pattern that applies and account for this pattern in the military capital estimates. Therefore, it is inappro-

⁶For a discussion of various types of sophisticated behavior in a competitive environment, see Franck (1979).

priate to make a separate adjustment for competitive obsolescence when determining the level of military capital. The measure of capital developed might best be interpreted as representing a situation in which the adversary's capital stock is held constant. This interpretation permits the direct comparison of one's own measure of aggregate capital with the corresponding measure of an adversary. Such a comparison provides a great deal of information about relative military capability.

V. CONCLUSION

This study has examined the determination of two aggregate measures of military capital. One measure, the instantaneous productive capacity of military capital K_I , is the monetary value of the services provided by the durable physical assets at a point in time. A second measure, the long-run productive capacity of military capital K_L , is the monetary value of the services provided by the assets over the remainder of their service lives.

Both measures are weighted aggregations of the number of units acquired each period in each asset class, $N^i(v)$. The instantaneous measure weights the acquisitions with the monetary value, measured at base period t_0 , of the level of effectiveness provided at t , $B^i(t_0)h^i(q^i)\Phi^i(t-v)$; the long-run measure weights the acquisitions with the corresponding demand price of the assets, $P^{id}(q^i, t_0, t-v)$. For the special case of exponential deterioration, these factors can be determined if military decisions are efficient and one knows the acquisition price function $P^i(q^i, t)$, the quality level $q^i(v)$, the age of each asset $t-v$, the service life λ^i , and the maintenance expenditure function $M^i(q^i, t, t-v)$. This is the type of information that is generated during the military planning, programming, and budgeting process. Therefore, the determination of the military capital measures is a feasible undertaking. Such an undertaking would be useful to a policymaker who needs summary measures of the different dimensions of military capability, relative military power, and the status of the long-term competition.

Appendix

STRAIGHT-LINE DETERIORATION

Section III concentrated on the use of the exponential deterioration function in military capital analysis. This deterioration pattern has a number of desirable properties against which any alternative deterioration assumption must be compared. This appendix derives the valuation functions for straight-line deterioration. It is shown that straight-line deterioration can be derived as a special case of exponential depreciation.

Under straight-line deterioration, the deterioration function is of the form

$$\Phi(t - v) = 1 - \omega(t - v), \quad t - v \leq \ell \quad (\text{A.1})$$

$$0, \quad t - v > \ell$$

where ω is the constant decline in relative efficiency that occurs each period as a result of the increase in the asset's age. This decline continues until the age of the asset equals the service life ℓ ; it is then withdrawn from the military capital stock.

As discussed in Sec. III, it is possible for equipment to be withdrawn from the inventory when $\Phi(t - v)$ is positive, because the asset is withdrawn when gross benefits of additional service life equal the additional maintenance costs.

One special case of Eq. (A.1) is known as "one-hoss shay" deterioration. This pattern occurs when the output produced by the military asset remains at its initial efficiency level until it is taken out of the inventory at ℓ . This situation might arise if the asset is maintained at its original capability level for the entire service life. If this pattern of deterioration occurs, the constant ω of Eq. (A.1) equals zero. This deterioration pattern is a special case of finite-life exponential deterioration. If the deterioration rate δ equals zero and the service life is ℓ , then the one-hoss shay case occurs. To simplify the discussion, it is assumed that the quality of the asset q , the acquisition price P , and the repair price R are constant over time. At t , the acquisition price is equated to the discounted value of the gross military benefits minus the total maintenance costs M of the new equipment:

$$P = B \int_t^{t+\ell} e^{-r(s-t)} [1 - \omega(s-t)] ds - M. \quad (\text{A.2})$$

This is the same form as Eq. (3.24) with the exception that the straight-line deterioration function is being used and the total maintenance cost is identified as M . As is the case for Eq. (3.24), the monetary value of an additional unit of military effectiveness B is assumed to be constant.

Evaluation of the integral of Eq. (A.2) yields

$$B = \frac{(P + M)r}{1 - e^{-r\ell}(1 - \omega\ell) - (\omega/r)(1 - e^{-r\ell})}, \quad (\text{A.3})$$

which can be rewritten in the equivalent form

$$B = (P + M)r + \frac{(P + M)r[e^{-rt}(1 - \omega l) + (\omega/r)(1 - e^{-rt})]}{1 - e^{-rt}(1 - \omega l) - (\omega/r)(1 - e^{-rt})} \quad (A.4)$$

A comparison of this expression with Eq. (3.5) shows that Eq. (A.4) applies to a situation in which there is an infinite service life and a *depreciation rate* $\tilde{\delta}$:

$$\tilde{\delta} = \frac{r[e^{-rt}(1 - \omega l) + (\omega/r)(1 - e^{-rt})]}{1 - e^{-rt}(1 - \omega l) - (\omega/r)(1 - e^{-rt})} \quad (A.5)$$

where the depreciation rate $\tilde{\delta}$ equals the exponential *deterioration rate* for an infinite service life situation that results in a user cost level equal to the right-hand side of Eq. (A.4). This means that there *is* an exponential process that yields a user cost level equivalent to the level obtained with straight-line deterioration.

For one-hoss shay deterioration, $\omega = 0$, and Eqs. (A.3) and (A.4) specialize to

$$B = \frac{(P + M)r}{1 - e^{-rt}} = (P + M)r + \frac{(P + M)re^{-rt}}{1 - e^{-rt}} \quad (A.6)$$

This is identical to Eq. (3.25) for the case in which the deterioration rate δ equals zero. One also obtains a depreciation rate equal to

$$\tilde{\delta} = \frac{re^{-rt}}{1 - e^{-rt}} \quad (A.7)$$

As B equals the monetary value of the service provided by the asset at a point in time, the demand price, $P^d(t - v)$, equals the discounted value of the services provided over the remaining life of the asset. For straight-line deterioration, the demand price equals

$$P^d(t - v) = B \int_t^{l+v} e^{-rs-t} \{1 - \omega[s + (t - v)]\} ds - M(t - v), \quad (A.8)$$

where $M(t - v)$ is the total remaining maintenance cost of an asset whose age is $t - v$. The evaluation of this integral yields

$$P^d(t - v) = B \{1 + [1 - \omega(t - v)] \left[\frac{1 - e^{-r[l - (t - v)]}}{r} \right] - (\omega/r^2) [e^{-r[l - (t - v)]} \{-r(l - (t - v) - 1)\}] \} - M(t - v), \quad (A.9)$$

where B is determined by Eq. (A.3). Even if maintenance expenditures $M(t - v)$ are ignored, the demand price does not decline linearly with asset age when this deterioration pattern is assumed: Straight-line *deterioration* is not the same as straight-line *depreciation*. For the one-hoss shay situation, the special case of Eq. (A.9) is

$$P^d(t - v) = \frac{(P + M)r}{1 - e^{-rt}} \left\{ 1 + \frac{1 - e^{-r[l - (t - v)]}}{r} \right\} - M(t - v), \quad (A.10)$$

which is not in general a linear function of the asset age: The value of the asset is not *depreciating* by a constant amount each period.

How significant is one-hoss shay deterioration? It might be argued that military equipment really doesn't deteriorate because of the required maintenance activities that keep the equipment at the original efficiency level. The asset is ultimately retired primarily because of rising maintenance costs. However, this deterioration pattern is a special case of finite-life exponential deterioration and one can use Eq. (3.27) to test whether the deterioration rate δ equals zero. Even if δ equals zero, as shown by Eq. (A.7) there is still a constant depreciation rate $\tilde{\delta}$ that applies to the one-hoss shay situation. The user-cost expression, Eq. (A.4), contains a term $(P + M)\tilde{\delta}$ that can be interpreted as a depreciation cost. Also, as discussed on p. 7, if the acquisitions of the asset are growing at a constant rate, one-hoss shay deterioration will eventually be characterized by an exponential deterioration process.

The discussion in this appendix might be viewed as providing additional support for the use of exponential deterioration. This deterioration pattern provides a type of null-hypothesis in military capital analysis. Although the available data might ultimately indicate that a more complex pattern applies, the assumption of exponential deterioration seems to be a good starting point.

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